

## BRST Cohomology and Nonlocal Conserved Charges

Nathan Berkovits<sup>1</sup>

*Instituto de Física Teórica, Universidade Estadual Paulista  
Rua Pamplona 145, 01405-900, São Paulo, SP, Brasil*

A relation is found between nonlocal conserved charges in string theory and certain ghost-number two states in the BRST cohomology. This provides a simple proof that the nonlocal conserved charges for the superstring in an  $AdS_5 \times S^5$  background are BRST-invariant in the pure spinor formalism and are  $\kappa$ -symmetric in the Green-Schwarz formalism.

September 2004

---

<sup>1</sup> e-mail: nberkovi@ift.unesp.br

## 1. Introduction

Maldacena's conjecture that  $d = 4$   $\mathcal{N} = 4$  super-Yang-Mills theory is dual to superstring theory in an  $AdS_5 \times S^5$  background has been difficult to prove since the perturbative descriptions of these two theories do not overlap. To obtain non-perturbative information about the two theories, one possible tool could be integrability and there have been various papers discussing this possibility both on the super-Yang-Mills side and on the superstring side.

On the superstring side, Bena, Polchinski and Roiban [1] constructed an infinite set of nonlocal conserved charges for the Green-Schwarz (GS) superstring in an  $AdS_5 \times S^5$  background<sup>2</sup>, suggesting an integrable structure. Vallilo [4] then constructed an analogous set of nonlocal conserved charges using the pure spinor formalism for the superstring in an  $AdS_5 \times S^5$  background [5]. These nonlocal charges for the superstring were related in [6] to a corresponding set of nonlocal charges on the super-Yang-Mills side.

In the GS formalism for the superstring, invariance under  $\kappa$ -transformations is crucial for determining the physical spectrum. For example, classical  $\kappa$ -symmetry is preserved in a curved background when the background satisfies the low-energy supergravity equations of motion, so onshell massless vertex operators must be  $\kappa$ -symmetric at least at the classical level. Although quantum  $\kappa$ -transformations and massive GS vertex operators are not yet understood, it is reasonable to expect that all physical GS states should be  $\kappa$ -symmetric. This would imply that the conserved charges for physical symmetries should also be  $\kappa$ -symmetric.

In the pure spinor formalism for the superstring, the role of  $\kappa$ -symmetry is replaced by BRST invariance. In this case, quantization and massive vertex operators are well-understood, and the physical spectrum is described by states in the BRST cohomology. So physical symmetries in the pure spinor formalism must be BRST-invariant.

Surprisingly, it has not been previously verified if the nonlocal conserved charges of [1] are  $\kappa$ -symmetric, or if the nonlocal conserved charges of [4] are BRST-invariant. This led some people (including this author) to conclude that the charges were not  $\kappa$ -symmetric and to question their physical significance. Due to the insistence of Witten [7] that these nonlocal charges should describe physical symmetries in analogy with the nonlocal charges

---

<sup>2</sup> These nonlocal charges were also independently found by Polyakov [2]. Similar nonlocal charges have been proposed in [3].

on the super-Yang-Mills side [6], the  $\kappa$ -symmetry and BRST invariance of the nonlocal charges of [1] and [4] were investigated.

As described in section 2, the existence of an infinite set of BRST-invariant nonlocal charges can be deduced from the absence of certain states in the BRST cohomology at ghost-number two. These ghost-number two states are  $f_{AB}^C h^A h^B$  where  $h^A$  are the BRST-invariant ghost-number one states associated with the global isometries and  $f_{AB}^C$  are the structure constants. Whenever  $f_{AB}^C h^A h^B$  can be written as  $Q\Omega^C$  for some  $\Omega^C$  (i.e. whenever  $f_{AB}^C h^A h^B$  is not in the BRST cohomology), one can construct an infinite set of BRST-invariant nonlocal charges. It would be interesting to know if quantum corrections to the ghost-number two cohomology are related to the potential anomalies discussed in [8].

In section 3 of this paper, it is shown using the pure spinor formalism for the superstring in an  $AdS_5 \times S^5$  background that the relevant ghost-number two states are absent from the classical BRST cohomology. The corresponding infinite set of BRST-invariant nonlocal charges is then explicitly constructed and shown to coincide with the conserved charges found by Vallilo in [4]. It is also shown that the conserved charges of Bena, Polchinski and Roiban in [1] are  $\kappa$ -symmetric.

## 2. Relation of Nonlocal Charges with BRST Cohomology

Suppose one has a BRST-invariant string theory with global physical symmetries described by the charges  $q^A = \int d\sigma j^A(\sigma)$ . Since these symmetries take physical states to physical states,  $q^A = \int d\sigma j^A(\sigma)$  must satisfy  $Q(q^A) = 0$  where  $Q$  is the BRST charge. Note that  $\{Q, b_0\} = H$  where  $H$  is the Hamiltonian, so BRST invariance implies charge conservation if  $q^A$  commutes with the  $b_0$  ghost, i.e. if  $q^A$  can be chosen in Siegel gauge. With the exception of the zero-momentum dilaton, it is expected that all ghost-number zero states in the BRST cohomology can be chosen in Siegel gauge.<sup>3</sup>

Since  $Q(\int d\sigma j^A(\sigma)) = 0$ ,  $Q(j^A) = \partial_\sigma h^A$  for some  $h^A$  of ghost-number one. And  $Q^2 = 0$  implies that  $Q(\partial_\sigma h^A) = 0$ , which implies that  $Q(h^A) = 0$  since there are no  $\sigma$ -independent worldsheet fields.

---

<sup>3</sup> In the pure spinor formalism for the superstring, there is no natural  $b$  ghost. Nevertheless, it is expected that for any ghost-number zero state in the pure-spinor BRST cohomology, there exists a gauge in which the state is annihilated by  $H$ .

Consider the BRST-invariant ghost-number two states  $f_{AB}^C : h^A h^B :$  where  $f_{AB}^C$  are the structure constants and normal-ordering is defined in any BRST-invariant manner, e.g.  $: h^A(z) h^B(z) : \equiv \frac{1}{2\pi i} \oint dy (y-z)^{-1} h^A(y) h^B(z)$  where the contour of  $y$  goes around the point  $z$ . It will now be shown that whenever  $f_{AB}^C : h^A h^B :$  is not in the BRST cohomology<sup>4</sup>, i.e. whenever there exists an operator  $\Omega^C$  satisfying  $Q(\Omega^C) = f_{AB}^C : h^A h^B :$ , one can construct an infinite set of nonlocal BRST-invariant charges.

To prove this claim, consider the nonlocal charge

$$k^C = f_{AB}^C : \int_{-\infty}^{\infty} d\sigma j^A(\sigma) \int_{-\infty}^{\sigma} d\sigma' j^B(\sigma') : .$$

Using  $Q(j^A) = \partial_{\sigma} h^A$ , one finds that  $Q(k^C) = \int d\sigma l^C(\sigma)$  where

$$l^C = -2f_{AB}^C : h^A(\sigma) j^B(\sigma) : .$$

One can check that  $Q(l^C) = f_{AB}^C \partial_{\sigma} (: h^A h^B :)$ , so  $Q(l^C - \partial_{\sigma} \Omega^C) = 0$  where  $\Omega^C$  is the operator which is assumed to satisfy  $Q(\Omega^C) = f_{AB}^C : h^A h^B :$ .

Since  $(l^C - \partial_{\sigma} \Omega^C)$  has +1 conformal weight and since BRST cohomology is only nontrivial at zero conformal weight,  $l^C - \partial_{\sigma} \Omega^C = Q(\Sigma^C)$  for some  $\Sigma^C$ . Using  $\Sigma^C$ , one can therefore construct the nonlocal BRST-invariant charge

$$\tilde{q}^C = f_{AB}^C : \int_{-\infty}^{\infty} d\sigma j^A(\sigma) \int_{-\infty}^{\sigma} d\sigma' j^B(\sigma') : - \int_{-\infty}^{\infty} d\sigma \Sigma^C(\sigma).$$

By repeatedly commuting  $\tilde{q}^C$  with  $\tilde{q}^D$ , one generates an infinite set of nonlocal BRST-invariant charges. So as claimed,  $f_{AB}^C : h^A h^B := Q(\Omega^C)$  implies the existence of an infinite set of nonlocal BRST-invariant charges.

### 3. BRST-Invariant Charges in $AdS_5 \times S^5$ Background

The results of the previous section will now be applied to the charges in an  $AdS_5 \times S^5$  background using the pure spinor formalism for the superstring. As in the Metsaev-Tseytlin GS action in an  $AdS_5 \times S^5$  background [10], the action in the pure spinor formalism [5] is constructed from left-invariant currents  $J^A = (g^{-1} \partial g)^A$  where  $g(x, \theta, \hat{\theta})$  takes values

---

<sup>4</sup> This BRST cohomology is defined in the “extended” Hilbert space which includes the zero mode of the  $x^m$  variables. As explained in [9], the inclusion of the  $x^m$  zero mode in the Hilbert space allows global isometries to be described by ghost-number one states in the cohomology.

in the coset  $PSU(2, 2|4)/SO(4, 1) \times SO(5)$ ,  $A = ([ab], m, \alpha, \hat{\alpha})$  ranges over the 30 bosonic and 32 fermionic elements in the Lie algebra of  $PSU(2, 2|4)$ ,  $[ab]$  labels the  $SO(4, 1) \times SO(5)$  “Lorentz” generators,  $m = 0$  to 9 labels the “translation” generators, and  $\alpha, \hat{\alpha} = 1$  to 16 label the fermionic generators. The action in the pure spinor formalism also involves left and right-moving bosonic ghosts,  $(\lambda^\alpha, w_\alpha)$  and  $(\hat{\lambda}^\alpha, \hat{w}_\alpha)$ , which satisfy the pure spinor constraints  $\lambda^\gamma \lambda^\gamma = \hat{\lambda}^\gamma \hat{\lambda}^\gamma = 0$ . These pure spinor ghosts transform as spinors under the local  $SO(4, 1) \times SO(5)$  transformations and couple to the  $AdS_5 \times S^5$  spin connection in the worldsheet action through their Lorentz currents  $N_{ab} = \frac{1}{2} w \gamma_{ab} \lambda$  and  $\hat{N}_{ab} = \frac{1}{2} \hat{w} \gamma_{ab} \hat{\lambda}$ .

To construct the nonlocal charges, the notation and conventions of [4] will be used where

$$\begin{aligned} J_0 &= (g^{-1} \partial g)^{[ab]} T_{[ab]}, & J_1 &= (g^{-1} \partial g)^\alpha T_\alpha, & J_2 &= (g^{-1} \partial g)^m T_m, \\ J_3 &= (g^{-1} \partial g)^{\hat{\alpha}} T_{\hat{\alpha}}, & N &= \frac{1}{2} (w \gamma^{[ab]} \lambda) T_{[ab]} \\ \bar{J}_0 &= (g^{-1} \bar{\partial} g)^{[ab]} T_{[ab]}, & \bar{J}_1 &= (g^{-1} \bar{\partial} g)^\alpha T_\alpha, & \bar{J}_2 &= (g^{-1} \bar{\partial} g)^m T_m, \\ \bar{J}_3 &= (g^{-1} \bar{\partial} g)^{\hat{\alpha}} T_{\hat{\alpha}}, & \hat{N} &= \frac{1}{2} (\hat{w} \gamma^{[ab]} \hat{\lambda}) T_{[ab]}, \\ \partial &= \frac{1}{2} \left( \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \sigma} \right), & \bar{\partial} &= \frac{1}{2} \left( \frac{\partial}{\partial \tau} - \frac{\partial}{\partial \sigma} \right), \end{aligned} \tag{3.1}$$

and  $T_A$  are the  $PSU(2, 2|4)$  Lie algebra generators. It will be convenient to also introduce the notation

$$\lambda = \lambda^\alpha T_\alpha, \quad \hat{\lambda} = \hat{\lambda}^\alpha T_{\hat{\alpha}}.$$

Note that  $\lambda$  and  $\hat{\lambda}$  are fermionic since  $(T_\alpha, T_{\hat{\alpha}})$  are fermionic and  $(\lambda^\alpha, \hat{\lambda}^\alpha)$  are bosonic.

Under classical BRST transformations generated by

$$Q = \int d\sigma (\lambda^\alpha J_3^\alpha + \hat{\lambda}^\alpha \bar{J}_1^\alpha) \delta_{\alpha\hat{\alpha}},$$

$g$  transforms by right-multiplication as

$$Q(g) = g(\lambda + \hat{\lambda}) \tag{3.2}$$

and the pure spinors transform as

$$Q(N) = -2[J_3, \lambda], \quad Q(\hat{N}) = -2[\bar{J}_1, \hat{\lambda}], \quad Q(\lambda) = Q(\hat{\lambda}) = 0.$$

The left-invariant currents therefore transform as

$$Q(J_j) = \delta_{j+3,0} \partial \lambda + [J_{j+3}, \lambda] + \delta_{j+1,0} \partial \hat{\lambda} + [J_{j+1}, \hat{\lambda}],$$

$$Q(\bar{J}_j) = \delta_{j+3,0} \bar{\partial} \lambda + [\bar{J}_{j+3}, \lambda] + \delta_{j+1,0} \bar{\partial} \hat{\lambda} + [\bar{J}_{j+1}, \hat{\lambda}],$$

where  $j$  is defined modulo 4, i.e.  $J_j \equiv J_{j+4}$ .

To prove the existence of an infinite set of BRST-invariant charges, one needs to find an  $\Omega = \Omega^C T_C$  satisfying  $Q\Omega =: \{h, h\}$  : where  $h = h^A T_A$ ,  $Q(j) = \partial_\sigma h$ , and  $q^A = \int d\sigma j^A$  are the charges associated with the global  $PSU(2, 2|4)$  isometries. It will be shown at the end of subsection (3.1) that  $Q(j) = \frac{1}{2} \partial_\sigma (g(\lambda - \hat{\lambda}) g^{-1})$ , so

$$h = \frac{1}{2} g(\lambda - \hat{\lambda}) g^{-1}. \quad (3.3)$$

Note that  $h$  is BRST-invariant since

$$Q(h) = \frac{1}{2} g\{(\lambda + \hat{\lambda}), (\lambda - \hat{\lambda})\} g^{-1} = \frac{1}{2} g((\lambda^\alpha \gamma_{\alpha\beta}^m \lambda^\beta) T_m - (\hat{\lambda}^\alpha \gamma_{\alpha\beta}^m \hat{\lambda}^\beta) T_m) g^{-1} = 0 \quad (3.4)$$

because of the pure spinor constraint. Consider the ghost-number two state

$$\{h, h\} = \frac{1}{2} g(\lambda - \hat{\lambda})(\lambda - \hat{\lambda}) g^{-1} = -\frac{1}{2} g\{\lambda, \hat{\lambda}\} g^{-1}. \quad (3.5)$$

Since  $\{\lambda + \hat{\lambda}, \lambda + \hat{\lambda}\} = 2\{\lambda, \hat{\lambda}\}$ , one can write this state as  $Q\Omega$  where

$$\Omega = -\frac{1}{4} g(\lambda + \hat{\lambda}) g^{-1}. \quad (3.6)$$

So  $Q\Omega = \{h, h\}$ , which implies the existence of an infinite set of BRST-invariant charges.

### 3.1. Explicit construction of BRST-invariant nonlocal charges

To explicitly construct these BRST-invariant charges, suppose one has a current whose  $\tau$ -component  $a$  satisfies

$$Qa = \partial_\sigma \Lambda + [a, \Lambda] \quad (3.7)$$

for some  $\Lambda$ . Then the charge

$$P(e^{-\int_{-\infty}^{\infty} d\sigma a(\sigma)}) \equiv \quad (3.8)$$

$$1 - \int_{-\infty}^{\infty} d\sigma a(\sigma) + \int_{-\infty}^{\infty} d\sigma a(\sigma) \int_{-\infty}^{\sigma} d\sigma' a(\sigma') - \int_{-\infty}^{\infty} d\sigma a(\sigma) \int_{-\infty}^{\sigma} d\sigma' a(\sigma') \int_{-\infty}^{\sigma'} d\sigma'' a(\sigma'') + \dots$$

satisfies  $Q(P(e^{-\int_{-\infty}^{\infty} d\sigma a(\sigma)})) = 0$ . So  $P(e^{-\int_{-\infty}^{\infty} d\sigma a(\sigma)})$  is a BRST-invariant charge.

To construct  $a$  satisfying (3.7), consider

$$a(c_j, \bar{c}_j) = g(c_0 N + c_1 J_1 + c_2 J_2 + c_3 J_3 + \bar{c}_0 \hat{N} + \bar{c}_1 \bar{J}_1 + \bar{c}_2 \bar{J}_2 + \bar{c}_3 \bar{J}_3) g^{-1} \quad (3.9)$$

where  $c_j$  and  $\bar{c}_j$  are constant coefficients. Note that  $a(c_j, \bar{c}_j)$  is invariant under the local  $SO(4, 1) \times SO(5)$  transformations.

Using the BRST transformations of (3.2),

$$\begin{aligned} Qa = & g[\lambda + \widehat{\lambda}, c_0 N + \bar{c}_0 \widehat{N} + \sum_{k=1}^3 (c_k J_k + \bar{c}_k \bar{J}_k)] g^{-1} \\ & + g(-2c_0[J_3, \lambda] - 2\bar{c}_0[J_1, \widehat{\lambda}] + c_1 \partial \lambda + c_3 \partial \widehat{\lambda} + \bar{c}_1 \bar{\partial} \lambda + \bar{c}_3 \bar{\partial} \widehat{\lambda}) g^{-1} \\ & + g \sum_{k=1}^3 (c_k [J_{k+3}, \lambda] + c_k [J_{k+1}, \widehat{\lambda}] + \bar{c}_k [\bar{J}_{k+3}, \lambda] + \bar{c}_k [\bar{J}_{k+1}, \widehat{\lambda}]) g^{-1}. \end{aligned} \quad (3.10)$$

And defining

$$\Lambda(b, \bar{b}) = g(b\lambda + \bar{b}\widehat{\lambda})g^{-1},$$

where  $b$  and  $\bar{b}$  are constant coefficients, one obtains

$$\begin{aligned} \partial_\sigma \Lambda + [a, \Lambda] = & g(b(\partial \lambda - \bar{\partial} \lambda) + \bar{b}(\partial \widehat{\lambda} - \bar{\partial} \widehat{\lambda})) g^{-1} + g\left[\sum_{j=0}^3 (J_j - \bar{J}_j), b\lambda + \bar{b}\widehat{\lambda}\right] g^{-1} \\ & + g[c_0 N + \bar{c}_0 \widehat{N} + \sum_{k=1}^3 (c_k J_k + \bar{c}_k \bar{J}_k), b\lambda + \bar{b}\widehat{\lambda}] g^{-1}. \end{aligned} \quad (3.11)$$

Setting (3.10) equal to (3.11) and using the worldsheet equations of motion [5][4]

$$\bar{\partial} \lambda + [\bar{J}_0, \lambda] = -\frac{1}{2}[\widehat{N}, \lambda], \quad \partial \widehat{\lambda} + [J_0, \widehat{\lambda}] = -\frac{1}{2}[N, \widehat{\lambda}], \quad (3.12)$$

and the pure spinor constraints  $[\lambda, N] = [\widehat{\lambda}, \widehat{N}] = 0$ , one obtains the conditions

$$c_1 = b, \quad -\bar{c}_3 = \bar{b}, \quad (3.13)$$

$$\begin{aligned} -c_1 + c_2 &= bc_1 + b, \quad -c_2 + c_3 = bc_2 + b, \quad -c_3 - 2c_0 = bc_3 + b, \\ -c_1 &= \bar{b}c_1 + \bar{b}, \quad -c_2 + c_1 = \bar{b}c_2 + \bar{b}, \quad -c_3 + c_2 = \bar{b}c_3 + \bar{b}, \quad 2c_0 + c_3 = -2\bar{b}c_0 + \bar{b}, \\ 2\bar{c}_0 + \bar{c}_1 &= -2b\bar{c}_0 - b, \quad -\bar{c}_1 + \bar{c}_2 = b\bar{c}_1 - b, \quad -\bar{c}_2 + \bar{c}_3 = b\bar{c}_2 - b, \quad -\bar{c}_3 = b\bar{c}_3 - b, \\ -\bar{c}_1 - 2\bar{c}_0 &= \bar{b}\bar{c}_1 - \bar{b}, \quad -\bar{c}_2 + \bar{c}_1 = \bar{b}\bar{c}_2 - \bar{b}, \quad -\bar{c}_3 + \bar{c}_2 = \bar{b}\bar{c}_3 - \bar{b}. \end{aligned}$$

The conditions of (3.13) are solved by

$$c_0 = \frac{1}{2}(1 - \mu^2), \quad c_1 = \pm \mu^{\frac{1}{2}} - 1, \quad c_2 = \mu - 1, \quad c_3 = \pm \mu^{\frac{3}{2}} - 1, \quad (3.14)$$

$$\bar{c}_0 = \frac{1}{2}(\mu^{-2} - 1), \quad \bar{c}_1 = 1 \mp \mu^{-\frac{3}{2}}, \quad \bar{c}_2 = 1 - \mu^{-1}, \quad \bar{c}_3 = 1 \mp \mu^{-\frac{1}{2}},$$

$$b = \pm \mu^{\frac{1}{2}} - 1, \quad \bar{b} = \pm \mu^{-\frac{1}{2}} - 1,$$

which coincides with the solution of [4] for conserved currents.

Note that the global charge  $q = \int d\sigma j(\sigma)$  can be obtained from (3.9) by expanding  $a(\mu)$  near  $\mu = 1$ . If  $\mu = 1 + \epsilon$ , one finds that  $a(\mu) = \epsilon j + \mathcal{O}(\epsilon^2)$  and  $\Lambda(\mu) = \epsilon h + \mathcal{O}(\epsilon^2)$  where  $Q(j) = h$ . So from the formulas

$$b(\mu) = \mu^{\frac{1}{2}} - 1 = \frac{1}{2}\epsilon + \mathcal{O}(\epsilon^2), \quad \bar{b}(\mu) = \mu^{-\frac{1}{2}} - 1 = -\frac{1}{2}\epsilon + \mathcal{O}(\epsilon^2),$$

one learns that

$$h = \lim_{\epsilon \rightarrow 0} \epsilon^{-1} \Lambda(\mu) = \frac{1}{2} g(\lambda - \hat{\lambda}) g^{-1}, \quad (3.15)$$

as was claimed in (3.3).

### 3.2. $\kappa$ -symmetry of nonlocal GS charges

Finally, it will be shown that the nonlocal GS conserved charges of [1] are  $\kappa$ -symmetric. In conformal gauge for the GS superstring, the  $\kappa$ -transformations of  $g$  and the  $\kappa$ -transformations of the left-invariant currents can be obtained from the BRST transformations of (3.2) by replacing  $\lambda$  with  $[\kappa, J_2]$  and replacing  $\hat{\lambda}$  with  $[\hat{\kappa}, \bar{J}_2]$  where  $\kappa = \kappa^{\hat{\alpha}} T_{\hat{\alpha}}$  and  $\hat{\kappa} = \hat{\kappa}^{\alpha} T_{\alpha}$ . This is the  $AdS_5 \times S^5$  version of the procedure adopted in [11] where  $\lambda^{\alpha}$  is replaced by  $\Pi^m(\gamma_m \kappa)^{\alpha}$  and  $\hat{\lambda}^{\hat{\alpha}}$  is replaced by  $\bar{\Pi}^m(\gamma_m \hat{\kappa})^{\hat{\alpha}}$ . In GS conformal gauge,  $\eta_{mn} J_2^m J_2^n = \eta_{mn} \bar{J}_2^m \bar{J}_2^n = 0$  and the  $\kappa$ -transformations are constrained to satisfy  $\kappa^{\hat{\alpha}} \bar{J}_1^{\alpha} \delta_{\alpha \hat{\alpha}} = \hat{\kappa}^{\alpha} J_3^{\hat{\alpha}} \delta_{\alpha \hat{\alpha}} = 0$  so that the  $h_{zz}$  and  $h_{\bar{z}\bar{z}}$  components of the worldsheet metric do not transform. Together with the GS equations of motion  $[J_2, \bar{J}_1] = 0$  and  $[\bar{J}_2, J_3] = 0$ , these conditions imply that

$$[\lambda, J_2] = [\hat{\lambda}, \bar{J}_2] = [\lambda, \bar{J}_1] = [\hat{\lambda}, J_3] = 0. \quad (3.16)$$

Using the current of (3.9) with  $c_0 = \bar{c}_0 = 0$ , one finds that (3.10) is equal to (3.11) if

$$c_1 = b, \quad -\bar{c}_3 = \bar{b}, \quad (3.17)$$

$$-c_1 + c_2 = bc_1 + b, \quad -c_3 = bc_3 + b,$$

$$-c_1 = \bar{b}c_1 + \bar{b}, \quad -c_2 + c_1 = \bar{b}c_2 + \bar{b}, \quad c_3 = \bar{b},$$

$$\begin{aligned}\bar{c}_1 &= -b, & -\bar{c}_2 + \bar{c}_3 &= b\bar{c}_2 - b, & -\bar{c}_3 &= b\bar{c}_3 - b, \\ -\bar{c}_1 &= \bar{b}\bar{c}_1 - \bar{b}, & -\bar{c}_3 + \bar{c}_2 &= \bar{b}\bar{c}_3 - \bar{b}.\end{aligned}$$

The conditions of (3.17) are solved by

$$\begin{aligned}c_1 &= \pm\mu^{\frac{1}{2}} - 1, & c_2 &= \mu - 1, & c_3 &= \pm\mu^{-\frac{1}{2}} - 1, \\ \bar{c}_1 &= 1 \mp \mu^{\frac{1}{2}}, & \bar{c}_2 &= 1 - \mu^{-1}, & \bar{c}_3 &= 1 \mp \mu^{-\frac{1}{2}}, \\ b &= \pm\mu^{\frac{1}{2}} - 1, & \bar{b} &= \pm\mu^{-\frac{1}{2}} - 1,\end{aligned}\tag{3.18}$$

which coincides with the conserved GS charges of [1].

**Acknowledgements:** I would especially like to thank Edward Witten for stressing that the nonlocal charges in an  $AdS_5 \times S^5$  background should be  $\kappa$ -symmetric. I would also like to thank Radu Roiban, Brenno Carlini Vallilo and Edward Witten for useful discussions and for reading the draft, CNPq grant 300256/94-9, Pronex 66.2002/1998-9, and FAPESP grant 99/12763-0 for partial financial support, and the Simons Workshop in Mathematics and Physics at SUNY at Stony Brook for their hospitality.

## References

- [1] I. Bena, J. Polchinski and R. Roiban, *Hidden Symmetries of the  $AdS_5 \times S^5$  Superstring*, Phys. Rev. D69 (2004) 046002, hep-th/0305116.
- [2] A.M. Polyakov, *Conformal Fixed Points of Unidentified Gauge Theories*, Mod. Phys. Lett. A19 (2004) 1649, hep-th/0405106.
- [3] G. Mandal, N.V. Suryanarayana and S.R. Wadia, *Aspects of Semiclassical Strings in  $AdS_5$* , Phys. Lett. B543 (2002) 81, hep-th/0206103
- [4] B.C. Vallilo, *Flat Currents in the Classical  $AdS_5 \times S^5$  Pure Spinor Superstring*, JHEP 0403 (2004) 037, hep-th/0307018.
- [5] N. Berkovits, *Super-Poincaré Covariant Quantization of the Superstring*, JHEP 0004 (2000) 018, hep-th/0001035;  
 N. Berkovits and O. Chandía, *Superstring Vertex Operators in an  $AdS_5 \times S^5$  Background*, Nucl. Phys. B596 (2001) 185, hep-th/0009168;  
 B.C. Vallilo, *One-Loop Conformal Invariance of the Superstring in an  $AdS_5 \times S^5$  Background*, JHEP 0212 (2002) 042, hep-th/0210064.
- [6] L. Dolan, C. Nappi and E. Witten, *A Relation Between Approaches to Integrability in Superconformal Yang-Mills Theory*, JHEP 0310 (2003) 017, hep-th/0308089.
- [7] E. Witten, private communication.
- [8] E. Abdalla, M. Gomes and M. Forger, *On the Origin of Anomalies in the Quantum Non-local Charge for the Generalized Non-linear Sigma Model*, Nucl. Phys. B210 (1982) 181.
- [9] A. Astashkevich and A. Belopolsky, *String Center of Mass Operator and its Effect on BRST Cohomology*, Commun. Math. Phys. 186 (1997) 109, hep-th/9511111.
- [10] R.R. Metsaev and A.A. Tseytlin *Type IIB Superstring Action in  $AdS_5 \times S^5$  Background*, Nucl. Phys. B533 (1998) 109, hep-th/9805028.
- [11] I. Oda and M. Tonin, *On the Berkovits Covariant Quantization of GS Superstring*, Phys. Lett. B520 (2001) 398, hep-th/0109051;  
 N. Berkovits, *Towards Covariant Quantization of the Supermembrane*, JHEP 0209 (2002) 051, hep-th/0201151.